The University of Georgia

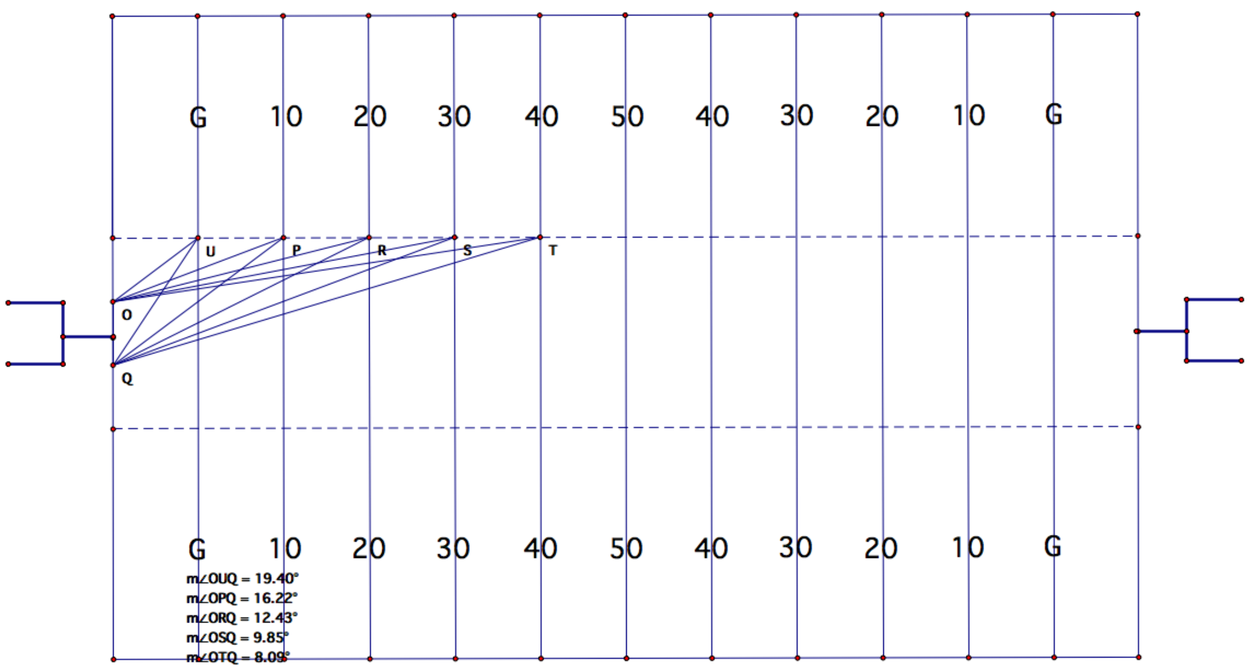
Department of Mathematics and Science Education
J. Wilson, EMAT 6680

EMAT 6680 - Assignment 6

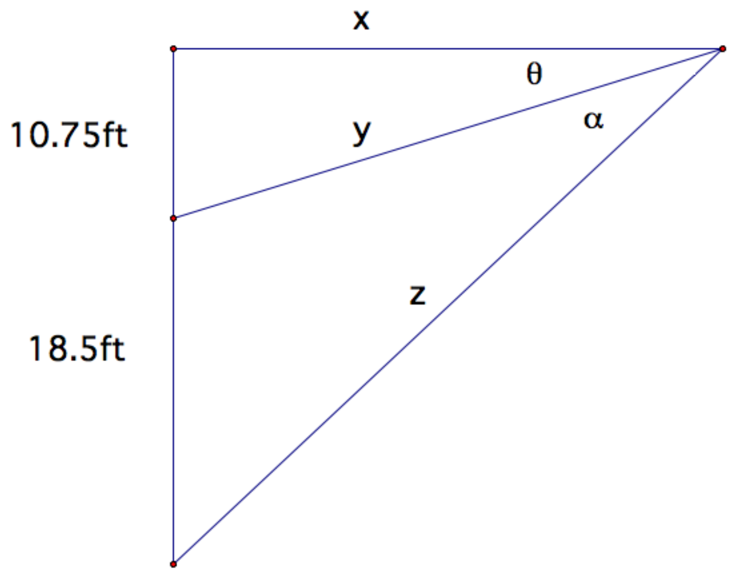
By Brandon Samples

Question: Find the optimal angle for a college football place kicker who must kick from the hash marks.

In order to understand what's going on, let's begin with model of a football field.

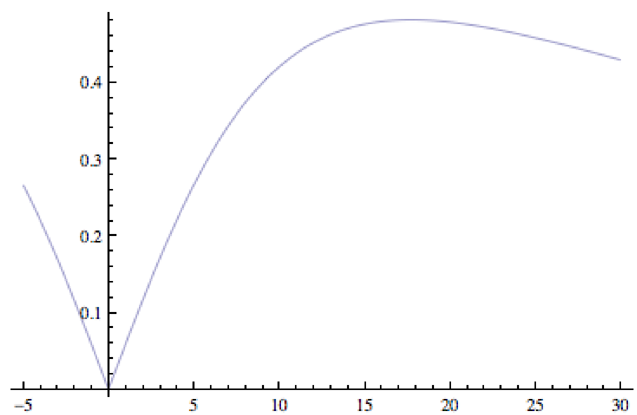


First, we can assume that the kicker makes his attempt from the right hash, since it will make no difference whether the kick is attempted from the right or the left side of the field. We have included a few kicks beginning at the goal line and then continuing up to the forty yard line in increments of ten yards. The angles included are estimates, of course, given that the field is not quite drawn to scale, although I attempted to draw everything as close as I could to the real thing. One thing to notice first off is that as the kicker moves backwards the angle decreases, so it would seem that we can put the commentators false claims to rest. Later on, we can prove this is true, so that it makes it harder when you back the football up five yards. Now, given a specific distance x, from the goal, we want to determine the maximum angle.



The angle as a function of x is given by the following function

$$\alpha(x) = \arccos\left(\frac{10.75^2 + 29.25^2 - 18.5^2 + 2x^2}{2\sqrt{x^2+10.75^2}\sqrt{x^2+29.25^2}}\right).$$



This is obtained by two applications of the Pythagorean Theorem. Now, by a little calculus $\alpha(x)$, we can maximize the given function by taking the derivative and setting it equal to zero. Appealing to Mathematica to avoid the unpleasant computation, we obtain:

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Solve[D[ArcCos[(10.75^2 + 29.25^2 - 18.5^2 + 2 x^2) / (2 Sqrt[x^2 + 10.75^2] Sqrt[x^2 + 29.25^2])], x] == 0,
 x]

{{x → -17.7324}, {x → 17.7324}}

x = 17.7325

17.7325

ArcCos[(10.75^2 + 29.25^2 - 18.5^2 + 2 x^2) / (2 Sqrt[x^2 + 10.75^2] Sqrt[x^2 + 29.25^2])]

0.480813

Out[9] x 180 / Pi

27.5485
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Specifically, the derivative equals zero, i.e., α takes on its maximum value when $x \approx 17.7324$ ft and gives α a value of 27.5845 degrees. This tells us that the optimal angle occurs at about the 6 yard line in the end zone. However, a kicker cannot kick from within the end zone, so we will say that the best place to kick occurs at the 7-8 yard line since a kicker lines up about 7-8 yards behind the line of scrimmage. Now, let's return to the discussion about the kicker taking a penalty in an effort to make the kick easier. Using a bit more of Mathematica, we see that the derivative is given by the following function along with the associated graph:

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In[3]:= D[ArcCos[(10.75^2 + 29.25^2 - 18.5^2 + 2 x^2) / (2 Sqrt[x^2 + 10.75^2] Sqrt[x^2 + 29.25^2])], x]
```

Out[3]=

$$-\frac{\frac{2x}{\sqrt{115.563+x^2}\,\sqrt{855.563+x^2}} - \frac{x\left(628.875+2x^2\right)}{2\sqrt{115.563+x^2}\,\left(855.563+x^2\right)^{3/2}} - \frac{x\left(628.875+2x^2\right)}{2\left(115.563+x^2\right)^{3/2}\sqrt{855.563+x^2}}}{\sqrt{1-\frac{\left(628.875+2x^2\right)^2}{4\left(115.563+x^2\right)\left(855.563+x^2\right)}}}$$

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In[10]:= Plot[Out[3], {x, -10, 330}, PlotRange -> {-.01, .01}]
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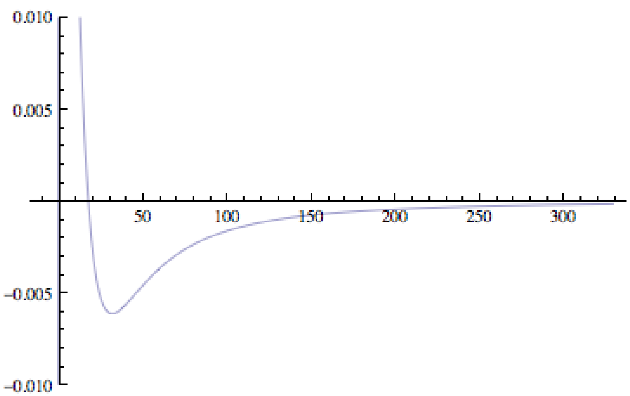
Out[10]=



Since the derivative is negative for all $x > 17.7324$, we see that the function is decreasing, which proves that the kicker should never take a penalty!